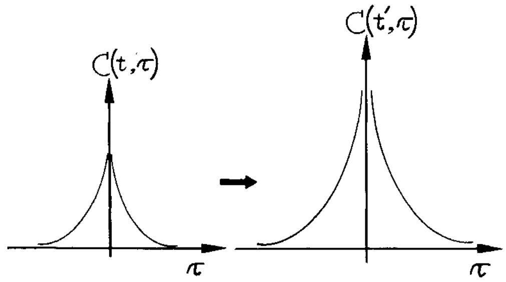

Fig. 1 "Representative" time evolution for "uniformly modulated" turbulence.

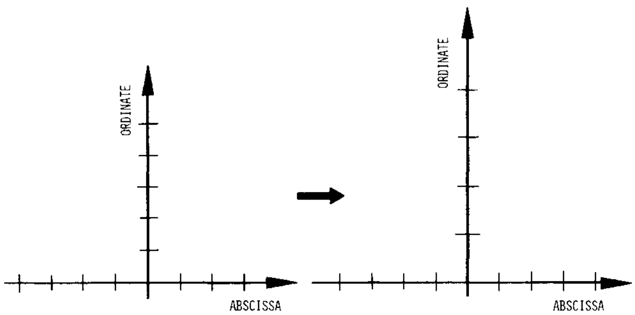

Fig. 2 Ordinate axis scale change effectively capturing the "uniformly modulated" structure.

Conclusions

Numerical results for this analysis are not available at this time since little is known about the functional form of $B(\omega_1,\omega_2)$ for atmospheric turbulence. This Note serves partly to introduce formally the applicability and advantages of bispectral concepts—concepts already employed in related fields[21-28] to aircraft response analyses. One unique feature of the bispectrum of clear importance in atmospheric turbulence is that it also *automatically* incorporates into its functional form the effects of the non-Gaussian statistical structure of the turbulence; the bispectrum for Gaussian turbulence is *identically* zero.

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Subcritical Damping Ratios of a Two-Dimensional Airfoil in Transonic Flow

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Introduction

THE *U-g* method gives critical flutter speeds that are in agreement with the traditional British approach with lined-up frequency parameters, but overestimates the relative damping ratio at other speeds.[1] However, useful values of

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damping can be obtained by using Frueh and Miller's[2] correction formula. Hassig[3] developed the *p-k* method which gives damping values in excellent agreement with the British method. This method iterates for the zeros of the flutter determinant using values of the aerodynamics interpolated from given values at a set of frequency parameters. It is assumed that, for sinusoidal motions with slowly varying amplitude, the aerodynamic forces can be approximated by those based on constant amplitude.

In this Note the high-frequency version of the ONERA unsteady transonic code[4] is used to compute lift and pitching moment coefficients of an NACA 64A006 airfoil oscillating in pitch and plunge. Comparisons of results from this code with LTRAN2-NLR[5] computations are given in Ref. 6 and, in general, are in quite good agreement at reduced frequencies up to, and, in some cases, exceeding, 0.4. With the aerodynamics data obtained from the ONERA code, the *U-g* method is used to compute critical flutter speeds and damping ratios based on Frueh and Miller's formula.[2] The results are compared with those from the *p-k* method.

Flutter Analysis of Two-Degree-of-Freedom Airfoil Motion

Figure 1 shows the notations used in the analysis of two-degree-of-freedom airfoil motion oscillating in pitch and plunge. The bending deflection is denoted by h, positive in the downward direction. α is the pitch angle about the elastic axis, positive with the nose up. The elastic axis is located at a distance $a_h b$ from the midchord, while the mass center is located at a distance $x_\alpha b$ from the elastic axis. Both distances are positive when measured toward the trailing edge of the airfoil. The aeroelastic equations of motion have been derived by Fung[7] and can be written as

$$\ddot{\xi} + x_\alpha \ddot{\alpha} + \omega_h^2 \xi = Q_h/mb \quad (1)$$

$$x_\alpha \ddot{\xi} + r_\alpha^2 \ddot{\alpha} + r_\alpha^2 \omega_\alpha^2 \alpha = Q_\alpha/mb^2 \quad (2)$$

where $\xi = h/b$ is the nondimensional displacement, m the mass per unit span of the airfoil, ω_h and ω_α the uncoupled plunging and pitching frequency, respectively, r_α the radius of gyration about the elastic axis, and Q_h and Q_α the total forces and moments acting about the elastic axis, respectively. In the absence of externally applied forces and moments, Q_h and Q_α can be written as

$$Q_h = -qc[C_{\ell_h}\xi/2 + C_{\ell_\alpha}\alpha] \quad (3)$$

$$Q_\alpha = -qc^2[C_{m_h}\xi/2 + C_{m_\alpha}\alpha] \quad (4)$$

where q is the dynamic pressure, C_ℓ and C_m the lift and moment coefficients with the subscripts h and α denoting unit plunging and pitching motions, respectively.

In the *U-g* method, a structural damping coefficient g is introduced into Eqs. (1) and (2) by multiplying the third term of

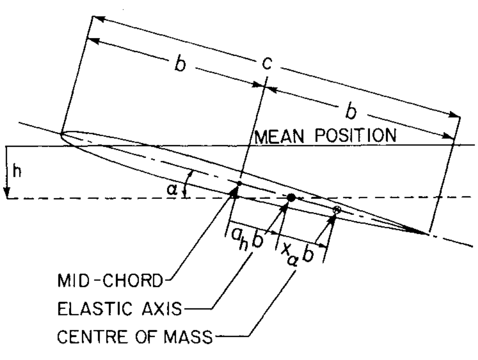

Fig. 1 Two-degree-of-freedom airfoil motion.

the two equations by the factor $(1+ig)$. For harmonic oscillations, ξ and α can be written as

$$\xi = \xi_0 e^{i\omega t} \quad (5)$$

$$\alpha = \alpha_0 e^{i\omega t} \quad (6)$$

Let $\mu = m/\pi b^2 \rho$ to be airfoil-air mass ratio, $k_c = \omega c/U$ the reduced frequency, and define

$$\lambda = \mu(1+ig)(\omega_\alpha^2 b^2/U^2) \quad (7)$$

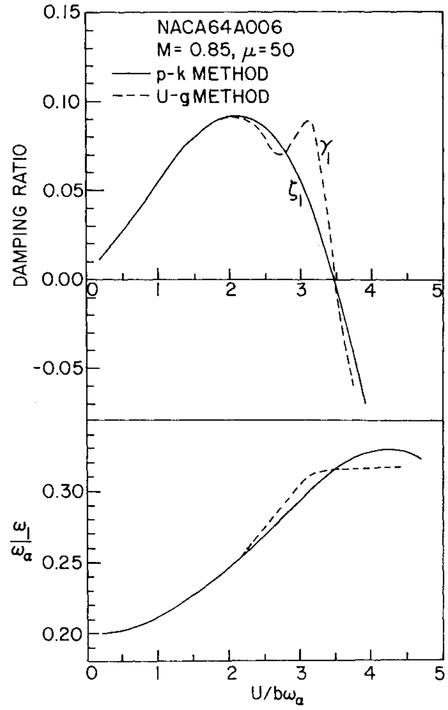

Fig. 2a Comparison of damping ratio and ω_1/ω_α with $U/b\omega_\alpha$ between the *U-g* and *p-k* methods for an NACA 64A006 airfoil at $M=0.85$ and $\mu=50$.

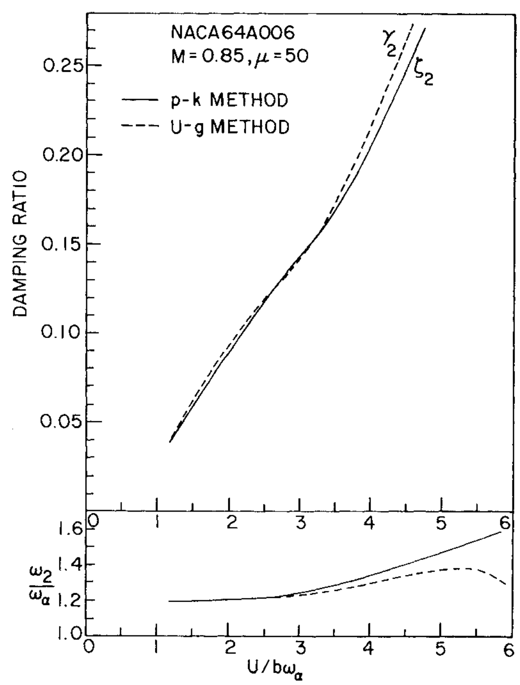

Fig. 2b Comparison of damping ratio and ω_2/ω_α with $U/b\omega_\alpha$ between the *U-g* and *p-k* methods for an NACA 64A006 airfoil at $M=0.85$ and $\mu=50$.

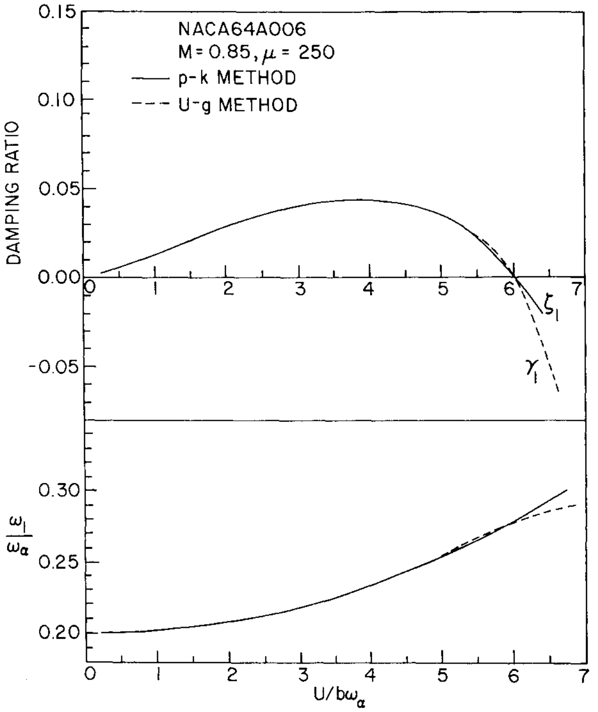

Fig. 3a Comparison of damping ratio and ω_1/ω_α with $U/b\omega_\alpha$ between the *U-g* and *p-k* methods for an NACA 64A006 airfoil at $M=0.85$ and $\mu=250$.

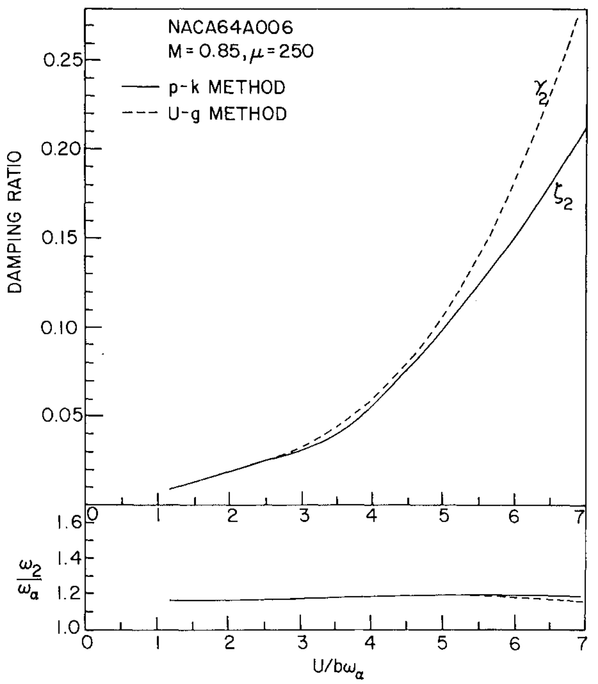

Fig. 3b Comparison of damping ratio and ω_2/ω_α with $U/b\omega_\alpha$ between the *U-g* and *p-k* methods for an NACA 64A006 airfoil at $M=0.85$ and $\mu=250$.

Upon substituting Eqs. (5-7) into Eqs. (1-4), the equations can be solved for the complex eigenvalue λ.

Hassig,[3] quoting from Frueh and Miller,[2] gives an expression for the damping ratio, which is written in the present notations as

$$\gamma = \frac{g}{2}\left[1 - \frac{2}{k_c}\,\frac{\mathrm{d}(\omega/\omega_\alpha)}{\mathrm{d}(U/b\omega_\alpha)}\right] \tag{8}$$

In the *p-k* method, Eqs. (5) and (6) are written in the following form:

$$\xi = \xi_0 e^{pt} \tag{9}$$

$$\alpha = \alpha_0 e^{pt} \tag{10}$$

where $p=\beta+i\omega$, and β is the damping factor. Substituting into Eqs. (1-4) yields a solution for p, and the damping ratio ζ is given as

$$\zeta = -\frac{\beta}{\sqrt{\beta^2+\omega^2}} \tag{11}$$

Damping Ratios for an NACA 64A006 Airfoil

The results for the damping ratios are presented for an NACA 64A006 airfoil at $M=0.85$. The following parameters are used: $x_\alpha=0.25$, $r_\alpha=0.5$, $\omega_h/\omega_\alpha=0.2$, and pitch axis $=0.25c$ ($a_h=-0.5$).

The effects of μ on the damping and frequency ratios are shown in Figs. 2 and 3 for $\mu=50$ and 250. The results for ζ and γ of the two modes are computed using Eqs. (8) and (11). The critical flutter speeds and frequencies obtained from the *U-g* and *p-k* methods are identical for all μ's considered. For the bending mode, the differences in subcritical damping and frequency ratios between the two methods decrease for increasing μ. At $\mu=250$, these two methods give almost identical results. For the torsion mode, the differences between ζ and γ increase with μ, while the differences in ω_2/ω_α decrease. The fairly good agreement between the two methods may be a special case for a two-degree-of-freedom, two-dimensional airfoil motion. Results for subcritical damping for more complex cases[1] show poor comparison between the *U-g* and British methods with lined-up frequency parameters which, in turn, has been demonstrated by Hassig[3] to be in good agreement with the *p-k* method.

Conclusions

For a two-dimensional airfoil oscillating in two degrees of freedom, the *p-k* method gives flutter speeds and k_c values identical to those from the *U-g* method. The computed subcritical damping ratios at $M=0.85$ for different μ show that the *U-g* method using Frueh and Miller's[2] formula gives results quite close to those from the *p-k* method, especially for large values of μ. The results are always in very close agreement for small values of $U/b\omega_\alpha$, irrespective of μ.

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